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ON BOUNDARIES AND INFLUENCES

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We prove an inequality relating the size of the boundary of a monotone subset of $\{0,1\}^n$ and the influences of the coordinates upon this set. It shows that if the boundary (resp. the influences) are small, the influences (resp. the boundary) are large.

1. Introduction

We will provide the set $\{0,1\}^n$ with the uniform probability μ (i.e. the normalized counting measure). For a point x in $\{0,1\}^n$, we denote $S_i(x)$ the point obtained from x by changing the i^{th} coordinate. Thus if $x=(x_j)_{j\le n}$, we have $S_i(x)=(y_j)_{j\le n}$ where $y_j=x_j$ for $j\ne i, y_i=1-x_i$. For a subset A of $\{0,1\}^n$, we define

$$A_i=\{x\in A; S_i(x)\notin A\}.$$

The number $I_i(A)=\mu(A_i)$ measures how much A depends upon the i^{th} coordinate, and is called the influence on A of the i^{th} coordinate. The boundary ∂A of A is the set of points in A that have a neighbor outside A, that is

$$\partial A=\bigcup_{i\le n} A_i.$$

In the present paper, we will consider only monotone sets, that is sets such that

$$x\in A, \forall i\le n, y_i\ge x_i \Rightarrow y\in A.$$

In that case, if $x\in A_i$ then $x_i=1$.

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In an important paper, G. A. Margulis [3] proved that for any subset A of $\{0,1\}^n$, the quantity

$$\mu(\partial A)\sum_{i\leq n} I_i(A) \tag{1.1}$$

is bounded below by a quantity depending only on $\mu(A)$, but not on n. This result already shows that ∂A and the influences cannot be too small at the same time.

While $\mu(\partial A)$ is an effective measure of the size of the boundary A, there is a more subtle way to measure the size of this boundary, which was introduced in [4].

It involves the function

$$h_A(x) = \operatorname{card}\{i \leq n; x \in A_i\},$$

that counts the "number of directions from which one can escape A". Observe that

$$\int h_A d\mu = \sum_{i\leq n} I_i(A), \tag{1.2}$$

so that, by Cauchy-Schwarz

$$\left(\int \sqrt{h_A} d\mu\right)^2 \leq \mu(\partial A)\sum_{i\leq n} I_i(A) \tag{1.3}$$

since $\partial A = \{h_A > 0\}$. It is proved in [4] that

$$\mu(A) \leq \frac{1}{2} \Rightarrow \int \sqrt{h_A} d\mu \geq \frac{1}{K}\mu(A)\sqrt{\log\frac{e}{\mu(A)}}. \tag{1.4}$$

Here, as well as in the rest of the paper, K denotes a universal constant, that may vary between occurrences. The isoperimetric-like inequality (1.4) makes the quantity $\int \sqrt{h_A} d\mu$ appear as a qualitatively correct measure of the size of the "surface area" of A. Our theorem relates this measure of surface with the size of the influences.

Theorem 1.1. *There exist numbers $\alpha, \beta > 0, \alpha \leq 1/2$, with the following properties. Consider, for $0 \leq x \leq 1$ the functions*

$$\varphi(x) = x(\log\frac{e}{x})^{\frac{1}{2}-\alpha}; \psi(x) = (\log\frac{e}{x})^{\alpha}. \tag{1.5}$$

Then for each monotone subset A of $\{0,1\}^n$, we have

$$\int \sqrt{h_A} d\mu \geq \beta\varphi(\mu(A)(1-\mu(A))\psi\left(\sum_{i\leq n} I_i^2(A)\right). \tag{1.6}$$

It is simple to see that $\sum_{i\leq n} I_i^2(A) \leq \mu(A)(1-\mu(A))$ (see (2.1)), so that Theorem 1.1 improves upon (1.4). When $\mu(A)=1/2$, (1.6) implies that

$$\int \sqrt{h_A}\, d\mu \geq \frac{1}{K}\psi\left(\sum_{i\leq n} I_i^2(A)\right) \tag{1.7}$$

so that $\int \sqrt{h_A}\, d\mu$ can be $O(1)$ only if $\sum_{i\leq n} I_i^2(A)$ is $\Omega(1)$. We believe actually that the sets of measure $1/2$ for which $\int \sqrt{h_A}\, d\mu$ is of order one are close to sets of the type $B_I = \{x; \sum_{i\in I} x_i \leq \frac{1}{2}\text{card } I\}$. One remarkable feature of Theorem 1.1 is that its formulation is "dimension independent", that is the sets B_I behave similarly with respect to (1.6) independently of card I.

The most obvious question left open by Theorem 1.1 is whether one can take $\alpha=1/2$. We conjecture that this is the case. This would be a very beautiful result. Indeed, if we set $M=\sup_{i\leq n} I_i(A)$ then $\sum_{i\leq n} I_i^2(A) \leq nM^2$, so using (1.3), (1.7) when $\mu(A)=1/2$ we would have

$$\sqrt{nM} \geq \left(\sum_{i\leq n} I_i(A)\right)^{1/2} \geq \frac{1}{K}\sqrt{\log \frac{1}{nM^2}}$$

and this implies $M \geq \frac{1}{K}\frac{\log n}{n}$, a fact proved in a remarkable paper of Kahn, Kalai, Linial [2]. The difficulty in proving this conjecture is as follows. The proof of [2] uses harmonic analysis, and we do not know how to take into account information about the boundary of A with this method. On the other hand, the proof of (1.4) in [4] uses a different approach (that will be used here) that does not seem powerful enough to prove the result of [2] (rather, it succeeds only in proving that $M \geq \frac{1}{K}(\log n)^\alpha / n$ for some $\alpha > 0$).

Apparently, the results presented here can be extended to the case where the uniform measure is replaced by a product measure $((1-p)\delta_0 + p\delta_1)^{\otimes n}$. But to do this we would have to write in the case $p \neq 1/2$ the proof of the lengthy Lemma 2.2 below, a task better left to the interested reader.

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2. Proof

Throughout the paper we will consider the Rademacher functions

$$r_i(x) = 2x_i - 1$$

on $\{0,1\}^n$. For a monotone set A, we have

$$I_i(A) = \int_A r_i d\mu.$$

The functions $(r_i)_{i\leq n}$ form an orthonormal system. Thus, for any set B, we have

$$\sum_{i\leq n}\left(\int_B r_i d\mu\right)^2 \leq \mu(B)(1-\mu(B)) \leq 1. \tag{2.1}$$

The following more precise fact is important. There, and through the paper, we simplify notation by setting

$$\ell(x) = \log\frac{e}{x}.$$

Lemma 2.1. *For some universal constant L, and for any subset B of $\{0,1\}^n$ we have*

$$\sum_{i\leq n}\left(\int_B r_i d\mu\right)^2 \leq L\mu(B)^2\ell(\mu(B)).$$

The elementary proof can be found in [5].

For a monotone set A, we will consider the quantity

$$t(A) = \sum_{i\leq n} I_i(A)^2.$$

The key to Theorem 1.1 is the following fact, that is proved in [5].

Lemma 2.2. *For some universal constant L, and any monotone subset A of $\{0,1\}^n$, we have*

$$\sum_{i\leq n}\sum_{j\neq i}\left(\int_{A_i} r_j\right)^2 \leq Lt(A)\ell(t(A)). \tag{2.2}$$

This inequality is better than what follows from Lemma 2.1, applied to all the sets A_i, and the improvement is crucial.

These lemmas represent the "harmonic analysis" part of the proof. We now turn to the "calculus part", the study of φ and ψ. The number α will be treated as a parameter, whose value will be determined later. The following is obvious from calculus.

Lemma 2.3. *The function* ψ *is convex and* $\psi'(t) = -\alpha\psi(t)/t\ell(t)$

In particular we have if $c \in [0,1]$

$$\psi(t) \geq \psi(c) - \alpha(t-c)\frac{\psi(c)}{c\ell(c)}. \tag{2.3}$$

Lemma 2.4. *We have* $\varphi' \geq 0, \varphi'(x) \leq \varphi(x)/x$, *and for* $x, b \in [0,1]$ *we have*

$$\varphi(x) \geq \varphi(b) + (x-b)\varphi'(b) - \frac{(x-b)^2\varphi(b)}{b^2\ell(b)}, \tag{2.4}$$

Proof. We have

$$\varphi'(x) = \ell(x)^{1/2-\alpha} - (\frac{1}{2} - \alpha)\ell(x)^{-1/2-\alpha} \geq 0$$

since $\ell(x) \geq 1$. Also, $\varphi'(x) \leq \ell(x)^{1/2-\alpha} = \varphi(x)/x$. Now, computation shows that

$$\varphi''(x) = -\frac{1}{x}\left(\frac{1}{2} - \alpha\right)\ell(x)^{-1/2-\alpha}\left(1 + \frac{\frac{1}{2}+\alpha}{\ell(x)}\right)$$

so that

$$0 \geq \varphi''(x) \geq -\frac{1}{x\ell(x)^{1/2+\alpha}} = -\frac{\varphi(x)}{x^2\ell(x)}. \tag{2.5}$$

We observe that the function $x \to x\ell(x)^{1/2+\alpha}$ increases on $[0,1]$, so that (2.4) follows from Taylor's formula for $x \geq b$. For $x \leq b$, we write

$$\varphi(x) - \varphi(b) - (x-b)\varphi'(b) = \int_x^b (t-x)\varphi''(t)dt.$$

We observe from (2.5) that if $t \leq b$

$$|\varphi''(t)| \leq bV/t$$

where $V = \varphi(b)/b^2\ell(b)$. Thus

$$\left|\int_x^b (t-x)\varphi''(t)dt\right| \leq V\int_x^b (t-x)\frac{b}{t}dt = bV\left[(b-x) - x\log\frac{b}{x}\right].$$

Now, $1 - t + t\log t \leq (1-t)^2$ since $\log t \leq t-1$. Using this for $t = x/b$ finishes the proof. ∎

We now set $f(x,t) = \varphi(x)\psi(t)$.

Lemma 2.5. *For $j=0,1$, consider $x_j=b+u_j$, $t_j=c+v_j$.*
Then

$$\frac{1}{2}f(x_0,t_0)f(x_1,t_1)\geq f(b,c)\Big[1-\frac{|u_0+u_1|}{b}-\frac{u_0^2+u_1^2}{b^2\ell(b)}$$
$$-\alpha\frac{|v_0+v_1|}{c\ell(c)}-\frac{\alpha|u_0v_0+u_1v_1|}{bc\ell(c)}-\frac{\alpha|u_0^2v_0+u_1^2v_1|}{b^2c\ell(b)\ell(c)}\Big].$$

Proof. Combining (2.3) and (2.4), we have

$$f(b+u,c+v)\geq\left(\varphi(b)+u\varphi'(b)-\frac{u^2\varphi(b)}{b^2\ell(b)}\right)\left(\psi(c)-\alpha v\frac{\psi(c)}{c\ell(c)}\right).$$

We write this for $(u,v)=(u_j,v_j)$, and we sum over $j=0,1$. We then use the bound $|\varphi'(b)|\leq\varphi(b)/b$. ∎

We now start the preparations for the induction argument. Consider a set $A\subset\{0,1\}^n$ $(n\geq 2)$. We set, for $j=0,1$

$$A^j=\{x\in\{0,1\}^{n-1};x^\frown j\in A\}.$$

Since A is monotone, $A^0\subset A^1$. Thus $A_n=(A^1\setminus A^0)\times\{1\}$. We denote by μ' the uniform measure on $\{0,1\}^{n-1}$. We set $a_j=\mu'(A^j)$, so $a=\mu(A)=\frac{1}{2}(a_0+a_1)$, $\mu(A_n)=I_n(A)=\frac{1}{2}(a_1-a_0)$.

We set

$$H(A)=\int\sqrt{h_A}\,d\mu.$$

Lemma 2.6. *We have*

$$H(A)-\frac{1}{2}(H(A^0)+H(A^1))\geq\frac{1}{8}\int_{A_n}\frac{1}{\sqrt{h_A}}d\mu. \tag{2.6}$$

Proof. Clearly we have

$$H(A)=\frac{1}{2}\left(\int_{A^1}\sqrt{h_{A^1}+1_{A^1\setminus A^0}}\,d\mu'+\int_{A^0}\sqrt{h_{A^0}}\,d\mu'\right)$$

so that the left hand side of (2.6) is at least

$$\frac{1}{2}\int_{A^1}\left(\sqrt{h_{A^1}+1_{A^1\setminus A^0}}-\sqrt{h_{A^1}}\right)d\mu'=\frac{1}{2}\int_{A^1\setminus A^0}\frac{1}{\sqrt{1+h_{A^1}}+\sqrt{h_{A^1}}}d\mu'$$
$$\geq\frac{1}{4}\int_{A^1\setminus A^0}\frac{1}{\sqrt{1+h_{A^1}}}d\mu'$$
$$\geq\frac{1}{8}\int_{A_n}\frac{1}{\sqrt{h_A}}d\mu.$$

since $h_A = 1 + h_{A^1}$ on A_n. ∎

In the induction argument, it will be essential to choose in an appropriate way "the last coordinate". This is the purpose of the next lemma. For a monotone set $A = \{0,1\}^n$, we set

$$d_i(A) = \sum_{j \neq i} \left(\int_{A_i} r_j \right)^2 .$$

Lemma 2.7. *For a monotone subset A of $\{0,1\}^n$ we can assume without loss of generality that*

$$\int_{A_n} \frac{1}{\sqrt{h_A}} d\mu \geq \frac{H(A)}{2} \left(\frac{\mu(A_n)^2}{t(A)} + \frac{d_n(A)}{Lt(A)\ell(t(A))} \right). \tag{2.7}$$

Proof. By (2.2) we have

$$\sum_{i \leq n} d_i(A) \leq Lt(A)\ell(t(A)).$$

By definition of $t(A)$, we have

$$\sum_{i \leq n} \mu(A_i)^2 = t(A).$$

Since $h_A = \sum_{i \leq n} 1_{A_i}$, we have

$$\sum_{i \leq n} \int_{A_i} \frac{1}{\sqrt{h_A}} d\mu = \int \sqrt{h_A} d\mu = H(A).$$

Thus

$$\sum_{i \leq n} \left(\int_{A_i} \frac{1}{\sqrt{h_A}} d\mu - \frac{H(A)}{2} \left(\frac{\mu(A_i)^2}{t(A)} + \frac{d_i(A)}{Lt(A)\ell(t(A))} \right) \right) \geq 0.$$

Hence, there must be one index i in the summation such that the corresponding term is ≥ 0. ∎

We now start the main argument, and show that, provided β, α are small enough, we can prove Theorem 1.1 by induction over n (certainly the theorem holds for $n = 1$). We perform the induction step from $n-1$ to n; we consider a monotone set $A \subset \{0,1\}^n$, and we assume that (2.7) holds. We keep the notation of Lemma 2.6. We set

$$b = a(1-a); \quad u_j = a_j(1-a_j) - b$$

$$c = t(A); \quad v_j = t(A^j) - c.$$

In order to use Lemma 2.5, we collect estimates. For simplicity, we write $d = d_n(A), \delta = a_1 - a_0$.

Lemma 2.8. *We have*

$$u_0 + u_1 = \delta^2/4 \tag{2.8}$$

$$|u_1 - u_0| \le \delta \tag{2.9}$$

$$|v_1 + v_0| \le 2d + 2\delta^2 \tag{2.10}$$

$$|v_1 - v_0| \le 4\sqrt{cd} \tag{2.11}$$

$$u_0^2 + u_1^2 \le \delta^2 \tag{2.12}$$

$$|u_0 v_0 + u_1 v_1| \le d\delta^2 + \delta^4 + 4\delta\sqrt{cd} \tag{2.13}$$

$$|u_0^2 v_0 + u_1^2 v_1| \le 2d\delta^2 + 2\delta^4 + \delta^3\sqrt{cd}. \tag{2.14}$$

Proof. We make no attempt to obtain sharp numerical values, but we use simple estimates instead. First, (2.8) and (2.9) are obvious. Next,

$$\begin{aligned} v_0 + v_1 &= (t(A^1) + t(A^0) - 2t(A)) \\ &= -2\delta^2 + \sum_{i<n}\left[\left(\int_{A^1} r_i d\mu'\right)^2 + \left(\int_{A^0} r_i d\mu'\right)^2 - \frac{1}{2}\left(\int_{A^1} r_i d\mu' + \int_{A^0} r_i d\mu'\right)^2\right] \\ &= -2\delta^2 + \frac{1}{2}\sum_{i<n}\left(\int_{A^1\setminus A^0} r_i d\mu'\right)^2 = -2\delta^2 + 2\sum_{i<n}\left(\int_{A_n} r_i d\mu\right)^2 \end{aligned}$$

To prove (2.11), we write

$$\begin{aligned} v_1 - v_0 &= \sum_{i<n}\left[\left(\int_{A^1} r_i d\mu'\right)^2 - \left(\int_{A^0} r_i d\mu'\right)^2\right] \\ &= \sum_{i<n}\left(\int_{A^0} r_i d\mu' + \int_{A^1} r_i d\mu'\right)\left(\int_{A^1\setminus A^0} r_i d\mu'\right) \\ &= 4\sum_{i<n}\int_A r_i d\mu \int_{A_n} r_i d\mu, \end{aligned}$$

and the result follows by Cauchy-Schwarz.

To prove the remaining estimates, we observe that

$$x_0 y_0 + x_1 y_1 = \frac{1}{2}\left((x_0 + x_1)(y_0 + y_1) + (x_0 - x_1)(y_0 - y_1)\right)$$

so that

$$|x_0 y_0 + x_1 y_1| \le \frac{1}{2}\left(|x_0 + x_1||y_0 + y_1| + |x_0 - x_1||y_0 - y_1|\right)$$

and we use (2.8) to (2.11). ∎

We now combine (2.6), the induction hypothesis, Lemmas 2.5 and 2.8 to get

$$H(A) \geq \frac{1}{8}\int_{B_n} \frac{1}{\sqrt{h_A}} d\mu + \beta f(b,c)\left[1 - \sum_{p\leq 5} R_p\right] \tag{2.15}$$

with

$$R_1 = \frac{1}{4}\frac{\delta^2}{b}; R_2 = \frac{\delta^2}{b^2\ell(b)}; R_3 = 2\alpha\frac{d+\delta^2}{c\ell(c)}$$
$$R_4 = \frac{\alpha}{bc\ell(c)}\left(d\delta^2 + \delta^4 + 4\delta\sqrt{cd}\right)$$
$$R_5 = \frac{\alpha}{b^2c\ell(c)\ell(b)}\left(2d\delta^2 + 2\delta^4 + \delta^3\sqrt{cd}\right)$$

and we want to show that $H(A) \geq \beta f(b,c)$.

Before this can be done, we have to identify the leading terms among $\sum_{p\leq 5} R_p$.

We observe that $x\ell(x) \leq 1$, so that $b \geq b^2\ell(b)$, and thus $4R_1 \leq R_2$. Also, we have $\delta^2 = (a_1 - a_0)^2 \leq 4I_n(A)^2 \leq 4c$, and, by Lemma 2.2, we have $d \leq Lc\ell(c)$. Thus, if α is small enough, we have

$$\alpha\frac{d+\delta^2}{c\ell(c)} \leq \frac{1}{6} \tag{2.16}$$

and thus

$$\frac{\alpha}{bc\ell(c)}(d\delta^2 + \delta^4) \leq R_1$$
$$\frac{\alpha}{b^2c\ell(c)\ell(b)}(2d\delta^2 + 2\delta^4) \leq R_2$$

Next, we observe that $\delta = a_1 - a_0 \leq 2\mu(A)$, and also $a_1 - a_0 \leq 2(1-\mu(A))$, so that $(a_1 - a_0)^2 \leq 4\mu(A)(1-\mu(A))$ i.e. $\delta^2 \leq 4b$. Thus

$$\frac{\delta^3\sqrt{cd}}{b^2c\ell(c)\ell(b)} \leq \frac{4\delta\sqrt{cd}}{bc\ell(c)}.$$

Using that $\sqrt{xy} \leq x + y$, we see that

$$\frac{\delta\sqrt{cd}}{bc\ell(c)} \leq \frac{\delta^2}{b^2\ell(c)} + \frac{d}{c\ell(c)}.$$

Now, by Lemma 2.1, $d \leq b$, so that $\ell(c) \geq \ell(b)$.

Combining these estimates, we deduce from (2.15) the more promising inequality

$$H(A) \geq \frac{1}{8}\int_{B_n} \frac{1}{\sqrt{h_A}}d\mu + \beta f(b,c)\left[1 - \frac{4\delta^2}{b^2\ell(b)} - \frac{3\alpha(d+\delta^2)}{c\ell(c)}\right]. \tag{2.17}$$

Since we want to show that $H(A) \geq \beta f(b,c)$, we can assume by contradiction that $H(A) \leq \beta f(b,c)$.

Case 1. We have

$$64\beta^2 f^2(b,c)\left[\frac{4}{b^2\ell(b)} + \frac{3\alpha}{c\ell(c)}\right] \leq 1 \tag{2.18}$$

In that case, we observe that by Cauchy-Schwarz, we have

$$\frac{1}{4}\delta^2 = \mu(A_n)^2 \leq \int_{A_n} \frac{1}{\sqrt{h_A}}d\mu \int_{A_n} \sqrt{h_A}d\mu \leq H(A)\int_{A_n} \frac{1}{\sqrt{h_A}}d\mu$$

so that

$$\frac{1}{16}\int_{A_n} \frac{1}{\sqrt{h_A}}d\mu \geq \frac{\delta^2}{64H(A)} \geq \frac{\delta^2}{64\beta f(b,c)}.$$

Using (2.18), it then follows from (2.17) that

$$H(A) \geq \frac{1}{16}\int_{A_n} \frac{1}{\sqrt{h_A}}d\mu + \beta f(b,c)\left[1 - \frac{3\alpha d}{c\ell(c)}\right]. \tag{2.19}$$

Using (2.16), we then see that $H(A) \geq \beta f(b,c)/2$. Now (2.7) implies

$$\int_{A_n} \frac{1}{\sqrt{h_A}}d\mu \geq \frac{1}{2}\frac{\beta d f(b,c)}{Lc\ell(c)}$$

Thus, if we take α small enough that $\frac{1}{32L} \geq 3\alpha$, we see that (2.19) implies $H(A) \geq \beta f(b,c)$.

Case 2. Then inequality (2.18) fails, which we rewrite as

$$64\beta^2 b^2 \ell(b)^{1-2\alpha}\ell(c)^{2\alpha}\left[\frac{4}{b^2\ell(b)} + \frac{3\alpha}{c\ell(c)}\right] \geq 1$$

and thus

$$256\beta^2\left[\left(\frac{\ell(c)}{\ell(b)}\right)^{2\alpha} + \alpha\frac{b^2}{c}\left(\frac{\ell(b)}{\ell(c)}\right)^{1-2\alpha}\right] \geq 1.$$

If α and β are small enough, it should be clear that this inequality implies $c \leq b^2/512$. Thus, since $\delta^2 \leq c$, we have $4\delta^2/b^2\ell(b) \leq 1/4$, and, recalling (2.16), we see from (2.17) that $H(A) \geq \beta f(b,c)/4$. We then appeal to (2.7) to see that (2.17) implies

$$H(A) \geq \beta f(b,c) \left[1 + \frac{\delta^2}{64c} + \frac{d}{64Lc\ell(c)} - \frac{4\delta^2}{b^2\ell(b)} - \frac{3\alpha(d+\delta^2)}{c\ell(c)}\right].$$

If $3\alpha \leq 1/64L, 3\alpha \leq 1/128$, this implies $H(A) \geq \beta f(b,c)$ and finishes the proof. ∎

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